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Antenna Radiation Modeling for Microwave Landing System

CONSTANTINE A. BALANIS, SENIOR MEMBER, IEEE, AND
YUK-BUN CHENG, STUDENT MEMBER, IEEE

Abstract—Geometric optics and diffraction techniques are used to develop radiation models of antennas mounted on an aircraft structure. Measurements at 35 GHz on a 1/35 scale model space shuttle and 1/11 scale Boeing 737 are used for comparison with computed patterns and are in very good agreement. Radiation coverage in the elevation plane on full scale Boeing 737 and Boeing 747 at 5.1 GHz, as applied to the microwave landing system (MLS), is examined for vertically- and horizontally-polarized antennas mounted at different locations on the aircraft.

I. INTRODUCTION

To meet the demands of an increased air traffic flow around airports and enhance the approach and landing capabilities of modern day high-speed aircraft, the Federal Aviation Administration in cooperation with international participation, is now developing a microwave approach and landing system better known as the microwave landing system (MLS) [1], [2]. This system will replace the existing lower frequency instrument landing system (ILS) now in operation at many airports around the country. The introduction of the MLS will improve the landing guidance accuracy especially during times of poor visibility.

One of the principal problems in the design of a safe, reliable, and accurate approach and landing system is the location of the antennas on the aircraft structure for desired radiation coverage and the selection of the best antenna polarization for efficient system operation. In the past, the usual design practice consisted of allocating specific locations to antennas regardless of electronic system requirements but based primarily upon structural specifications. Because of the complexity of the structural configuration of the airframe, analytical investigations for

system improvements were not encouraged and due to economic considerations extensive experimental studies were prohibitive. With the recent advances of the geometrical theory of diffraction and its application to many previously intractable problems [3], a mathematical modeling of the radiation mechanism of antennas on aircraft is now possible [4]–[9]. To improve the antenna system performance in the presence of the airframe for desired MLS coverage, extensive analytical and experimental investigations have been undertaken with some discussion and results presented in this paper. The aircraft modeling is two-dimensional with its structure approximated by sharp wedges [5], [6] as compared to the curved-surface modeling reported in [7]–[9]. Divergence factors are included to account for the radius of curvature of the structure in the transverse direction to the principal plane. Although the modeling is more accurate for electrically large bodies and it is presently limited to principal plane patterns, it is much simpler and more versatile to apply. Where as it would be difficult to model a concave surface, such as the nose-cockpit section, using curved-surface techniques, it would be much more attractive and simpler to use wedged-surface modeling. Also window openings and diffractions from their surrounding structures can easily be accounted for by wedge diffraction mechanisms.

Since the newly developed MLS system is primarily a landing system, its primary desired coverage is in the forward (nose) direction. It is then important to select the antenna type, polarization, and location which will maximize the coverage for the landing system and minimize the interference with other onboard avionics and structural obstacles such as the landing gear, wings, and stabilizers.

II. THEORY

To demonstrate the wedged-surface modeling technique, the method will be applied to antenna position 1 (station 222) of the Boeing 737 geometry shown in Fig. 1(a). Similar analysis is used for antenna position 2 (station 220) above the cockpit, as well as other positions, but will not be included here as it can be found in [10]. Computed and measured patterns for locations 1 and 2 will be presented along with data on a space shuttle model. A wedged-surface modeling of the Fig. 1(a) airplane configuration in the elevation plane is shown in Fig. 1(b). In addition to the direct radiation from the source toward the observation point, energy from the source directed toward wedges #1 and #2 undergoes diffraction. The field is scattered in all directions according to the laws of diffraction [11], [12] and its distribution is represented by diffraction coefficients [12], [13]. Diffracted energy from wedges #1 and #2 is directed toward wedges #3 and #4, respectively, where it undergoes an additional diffraction. This process continues indefinitely but, as it has been demonstrated in many previous problems [3], [4], first-order diffractions need only be included to obtain data within measuring accuracies. For the axial slot the energy from the source toward wedges #1 and #2 is very weak because of the vanishing boundary conditions on the tangential components of the E -field. In this paper the computed pattern for the axial slot in location 1 is that when the aperture is mounted on an infinite ground plane. However, if the slot is very near the wedge, such that the field at the wedge is not very weak, slope diffraction techniques [14] can be used to account for the diffraction. In order not to belabor the formulation and lose sight of the problem, the expressions for the most dominant field contributions will be outlined below. The methodology has well been explored and applied in the past by many investigators [3], [7], [12], [14], [15]. The details of the

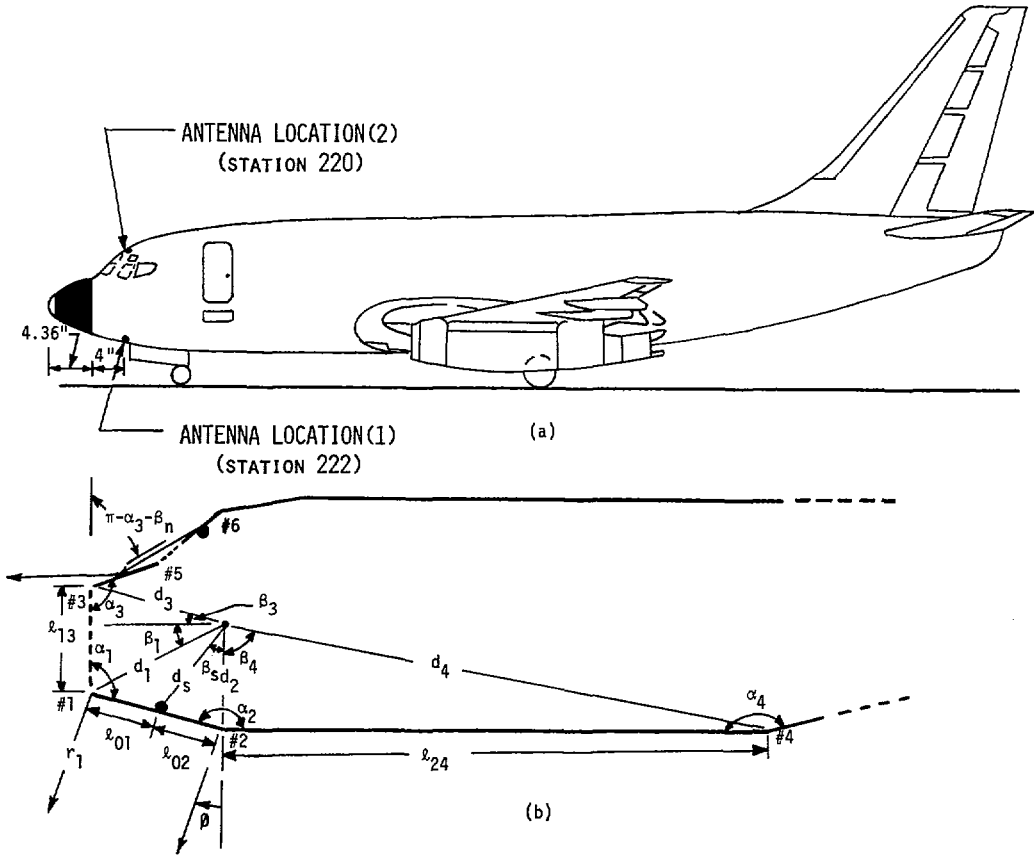


Fig. 1. Boeing 737 airplane configuration with two antenna locations. (a) 1/11 scale model. (b) wedged-surface modeling.

analysis for locations 1 and 2, as well as for other configurations, can be found in [10]. The airplane geometry in the transverse direction is assumed to be circular and the e^{-jkr}/r factor will be suppressed.

The radiation from the source is given by the following.

Direct Radiation

$$E_D(\phi) = R_D(\phi)e^{+jkd_s \cos(\beta_s - \phi)}, \quad \begin{cases} 5\pi/2 - \alpha_2 < \phi < 2\pi, \\ 0 < \phi < 3\pi/2 - \alpha_2 \end{cases} \quad (1)$$

where $R_D(\phi)$ is the far-zone field distribution of the antenna when mounted on an infinite ground plane.

Energy from the source directed toward wedge #1 undergoes diffraction. Its field contribution can be written as [12]

$$E_{D1}(\phi) = R_D(\phi = 3\pi/2 - \alpha_2) \frac{e^{-jk l_{01}}}{l_{01}} \sqrt{L} e^{+jkL} A(r_1) e^{-jkr_1}$$

$$R_{D1} = \begin{cases} R_{D1}, & 0 < \phi < \pi \\ R_{D1}', & \pi + \alpha_1 < \phi < 2\pi \end{cases} \quad (2)$$

$$R_{D1} = V_B(l_{01}, \pi - \alpha_1 + \phi, n_1) \quad (2a)$$

$$R_{D1}' = V_B(l_{01}, \phi - \alpha_1 - \pi, n_1) \quad (2b)$$

where $V_B(r, \psi, n)$ is the diffraction function of a two-dimensional conducting wedge [12], [13], L is a distance parameter, and $A(r_1)$ is a spatial attenuation which describes how the field intensity varies along the diffracted ray.

For spherical wave incidence normal to the edge of the wedge, L and $A(r_1)$ are given by [12]

$$L = \frac{l_{01} r_1}{r_1 + l_{01}} \quad (3)$$

$$A(r_1) = \sqrt{\frac{\rho_1}{r_1(\rho_1 + r_1)}} \quad (4)$$

where ρ_1 is the distance between the caustic at the edge and a second caustic formed by the diffracted rays. For $r_1 \gg \rho_1$, L and $A(r_1)$ reduce to

$$L = l_{01} \quad (5)$$

$$A(r_1) = \frac{\sqrt{\rho_1}}{r_1} \quad (6)$$

Thus we can simplify (2) and rewrite it, assuming far-field observations and suppressing the e^{-jkr}/r factor, as the following.

Diffractions from Wedge 1

$$E_{D1}(\phi) = R_D(3\pi/2 - \alpha_2) F_1 \begin{cases} V_B(l_{01}, \pi - \alpha_1 + \phi, n_1), & 0 < \phi < \pi \\ V_B(l_{01}, \phi - \alpha_1 - \pi, n_1), & \pi + \alpha_1 < \phi < 2\pi \end{cases} \quad (7)$$

$$F_1 = \sqrt{\frac{\rho_1}{l_{01}}} e^{+jkd_1 \cos(\pi/2 - \beta_1 - \phi)} \quad (7a)$$

$$\frac{1}{\rho_1} = \frac{1}{l_{01}} + \frac{2(\sin \alpha_2 + \cos \phi)}{l_{13}} \quad (7b)$$

where wedge #1 has been assumed to be a half plane ($n_1 = 2$). For $\pi < \phi < \pi + \alpha_1$, the field is trapped within the fuselage and its reradiation is of secondary order and has been neglected. In a similar manner the diffractions from the remaining wedges can be written as the following.

Diffractions from Wedge 3

$$E_{D3}(\phi) = R_D(3\pi/2 - \alpha_2)V_B(l_{01}, 2\pi - \alpha_1, n_1)F_3 \cdot [V_B(l_{13}, \phi, n_3) + V_B(l_{13}, \phi + 2\alpha_3, n_3)],$$

$$0 < \phi < 2\pi - \alpha_3 - \beta_n \quad (8)$$

$$F_3 = \sqrt{\frac{\rho_{13}}{l_{01}}} \sqrt{\frac{\rho_3}{l_{13}}} e^{+jkd_3 \cos(\phi - \beta_3 - \pi/2)} \quad (8a)$$

$$\frac{1}{\rho_{13}} = \frac{1}{l_{01}} + \frac{2(\sin \alpha_2 - 1)}{l_{13}} \quad (8b)$$

$$\frac{1}{\rho_3} = \frac{1}{l_{13}} - \frac{2(1 + \cos \phi)}{l_{13}} \quad (8c)$$

where wedge #3 has been assumed to be a half plane ($n_3 = 2$).

Diffractions from Wedge 2

$$E_{D2}(\phi) = R_D(5\pi/2 - \alpha_2)F_2 \begin{cases} V_B(l_{02}, 3\pi/2 - \alpha_2 - \phi, n_2), & 0 < \phi < 3\pi/2 - \alpha_2 \\ V_B(l_{02}, 7\pi/2 - \alpha_2 - \phi, n_2), & 3\pi/2 < \phi < 2\pi \end{cases} \quad (9)$$

$$F_2 = \sqrt{\frac{\rho_2}{l_{02}}} e^{+jkd_2 \cos \phi} \quad (9a)$$

$$\frac{1}{\rho_2} = \frac{1}{l_{02}} - \frac{(\sin \alpha_2 - \cos \phi)}{d_2} \quad (9b)$$

$$n_2 = (2 - \alpha_2/\pi). \quad (9c)$$

Diffractions from Wedge 4

$$E_{D4}(\phi) = R_D(5\pi/2 - \alpha_2)V_B(l_{02}, 2\pi - \alpha_2, n_2)F_4 \cdot \begin{cases} V_B(l_{24}, \pi/2 - \phi, n_4), & 0 < \phi < \pi/2 \\ V_B(l_{24}, 5\pi/2 - \phi, n_4), & \pi/2 + \alpha_4 < \phi < 2\pi \end{cases} \quad (10)$$

$$F_4 = \sqrt{\frac{\rho_{24}}{l_{02}}} \sqrt{\frac{\rho_4}{l_{24}}} e^{+jkd_4 \cos(\phi + \beta_4)} \quad (10a)$$

$$\frac{1}{\rho_{24}} = \frac{1}{l_{02}} - \frac{\sin \alpha_2}{d_2} \quad (10b)$$

$$\frac{1}{\rho_4} = \frac{1}{l_{24}} + \frac{\cos \phi}{d_2} \quad (10c)$$

$$n_4 = (2 - \alpha_4/\pi). \quad (10d)$$

Similar expressions have been derived for diffractions from wedges #5 and #6 and were included in the computations. They are not repeated here because of space limitations. It should also be pointed out that the divergence factors are not valid in the regions where the caustic distances, as defined by (7b), (8c), (9b), and (10c), become infinite. In these regions the divergence factor has been replaced by the first term of (7b), (8c), (9b), and (10c) which is a good approximation especially for bodies with large electrical dimensions.

The same techniques were used to develop radiation models of antennas mounted on position 2 (above the cockpit) of Boeing 737, as shown in Fig. 1(a), and space shuttle below the nose which will be discussed in the results. For the location 2 antenna on the Boeing 737, the modeling also includes an additional radiation mechanism to account for reflection from the upper part of the nose surface [10] as shown in Fig. 1(b).

III. COMPUTATIONS AND RESULTS

Figs. 2(a)–(d) include the measured and computed patterns of a circumferential aperture, axial aperture, quarter wavelength monopole, and short monopole located beneath the nose (station 222) of a 1/11 scale model Boeing 737, as shown on the insert. The frequency of operation was 35 GHz. The mathematical analysis was based on a two dimensional wedged-surface model outlined by (1)–(10d). The agreement between theory and experiment is very good except toward the upper part of the nose ($90^\circ < \phi < 180^\circ$). The mathematical simulation assumed that the radome part of the nose, shown darkened on the insert, was truncated. However, in the model used for the measurements, the radome part of the nose was left unpainted where the remaining structure was covered with conductive paint. It would then be expected that diffracted energy from the wedge, formed by the lower part of the truncated nose, will be different in the two cases and part of it will be trapped within the radome structure in the experimental model. It is encouraging, however, that the comparison is quite good even with the dissimilarity in the radome structure modeling. It is quite apparent from the patterns that the antennas operating in the vertical polarization mode in the elevation plane Fig. 2 (a), (c), (d) provide a much wider coverage toward the nose than the horizontally-polarized antenna Fig. 2(b).

In Figs. 3(a)–(d) the measured and computed patterns of the same antennas as in Figs. 2(a)–(d) are shown, this time mounted on a 1/35 scale model space shuttle. The antenna position was again beneath the nose, as shown on the insert, and the frequency of operation was 35 GHz. The orbiter configuration and some pertinent dimensions are shown in Fig. 4. The radome part of the nose, shown dashed in Fig. 4, was removed (truncated) to better simulate the dielectric radome configuration. The mathematical modeling used for the computation is similar to that developed for the Boeing 737 and outlined by (1)–(10d). As with the Boeing 737 patterns shown in Figs. 2(a)–(d), the vertical polarization patterns do exhibit wider coverage in the forward direction.

Another antenna location examined was that denoted as 2 in Fig. 1(a). This location is above the cockpit and it is very promising because of the absence of any structural features on the air-plane which would degrade its performance during landing, as would be the case for antennas mounted below the nose. Computed and measured patterns for elevation plane vertically- and horizontally-polarized apertures are shown in Figs. 5 and 6. The agreement between theory and experiment is very good for the null structure for the vertically-polarized aperture toward the forward (nose) region is due to the modeling of the radome-nose interface in order to make it analytically tractable. Where for the measurements the radome part of the nose, shown darkened in Fig. 1(a), was present, in the analytical modeling it was assumed to be truncated. It is then expected, because of the absence of the radome, that more energy will be allowed to radiate toward the region below the nose. Also the diffractions from the wedge formed at the radome-nose interface are quite strongly excited,

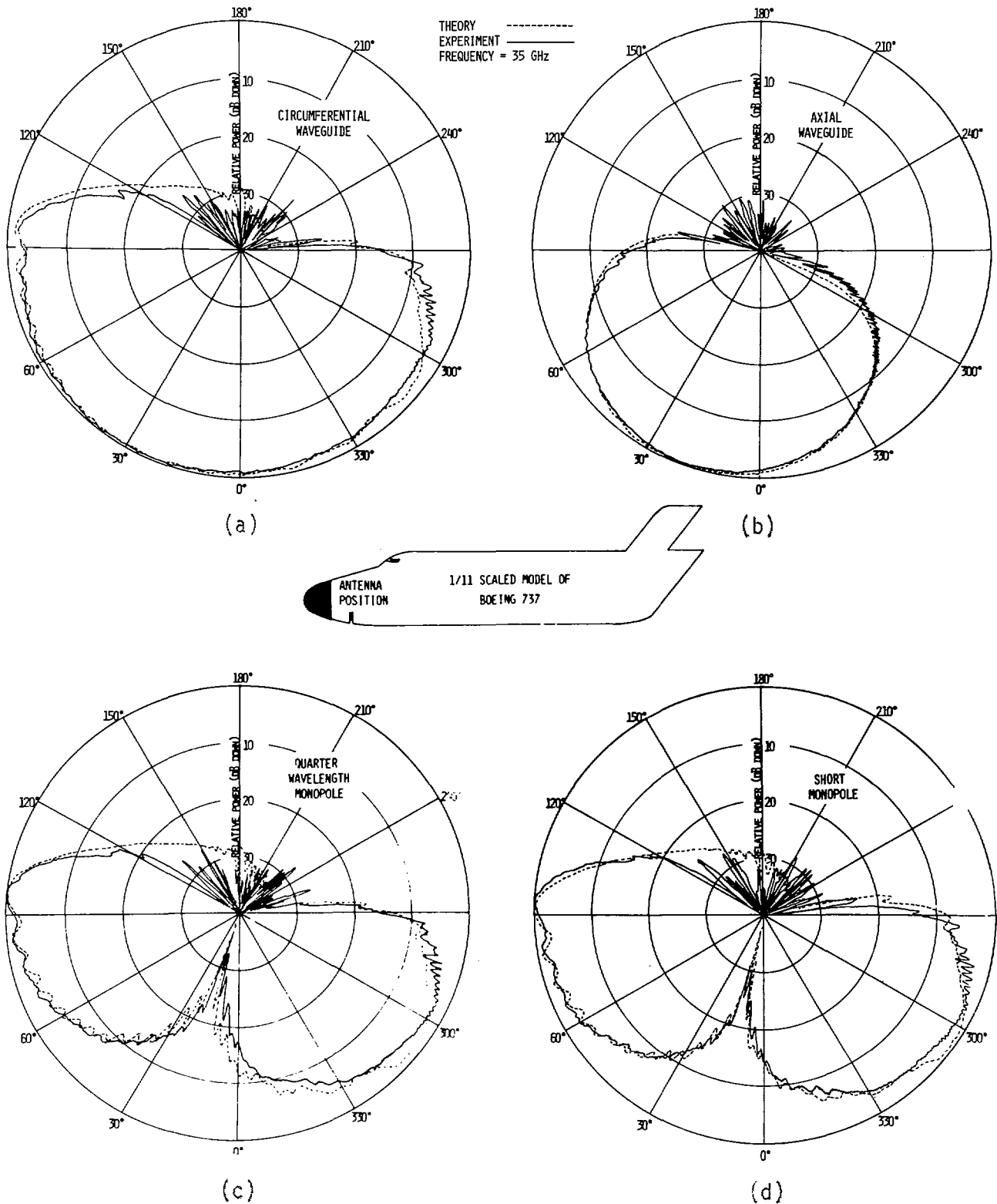


Fig. 2. Measured and predicted patterns of antennas located at bottom of nose (station 222) of 1/11 scale model Boeing 737.

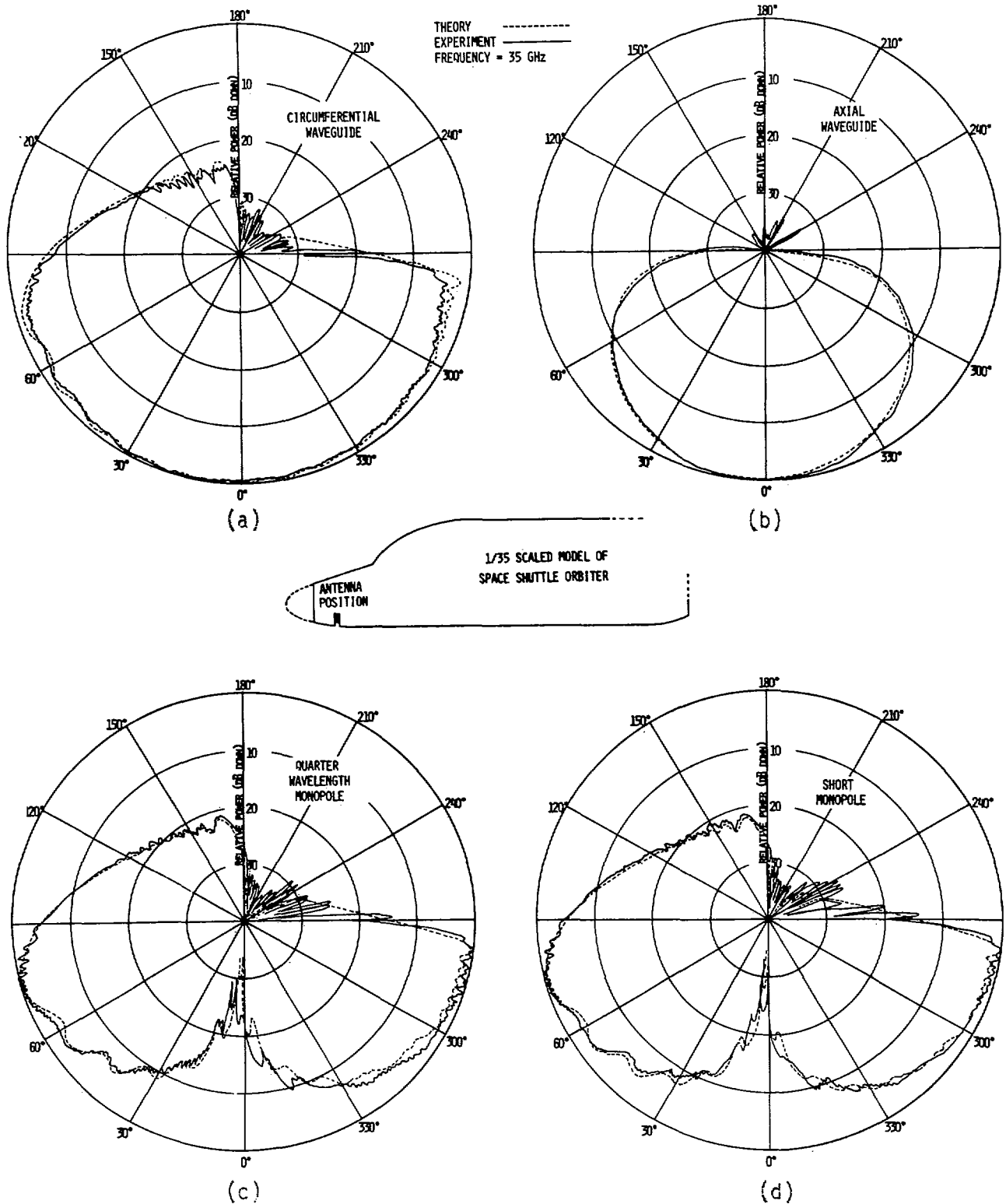


Fig. 3. Measured and predicted patterns of antennas located at bottom of nose of 1/35 scale model space shuttle orbiter.

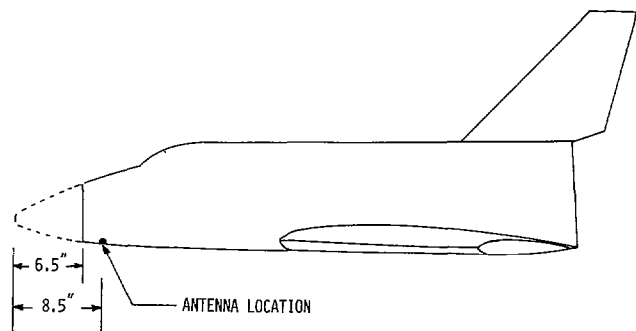


Fig. 4. Space shuttle orbiter configuration with dimensions and antenna location on 1/35 scale model.

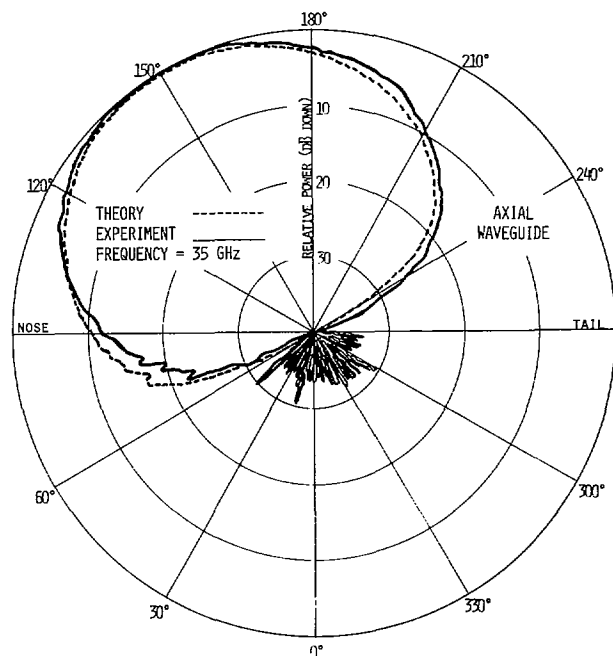


Fig. 6. Elevation plane amplitude patterns of axial slot at location 2 (station 220) of 1/11 scale model Boeing 737 shown in Fig. 1(a).

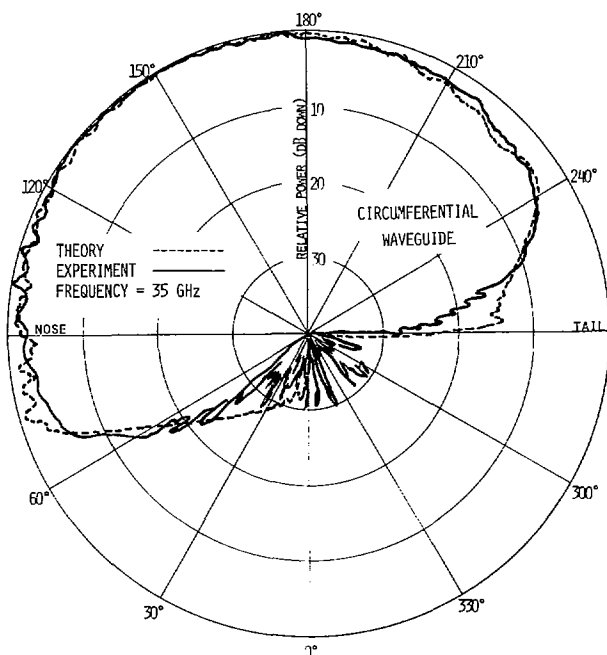


Fig. 5. Elevation plane amplitude patterns of circumferential slot at location 2 (station 220) of 1/11 scale model Boeing 737 shown in Fig. 1(a).

especially in the forward direction, and do interfere with the direct radiation to form the null and peaks demonstrated in Fig. 5. The disagreements toward the tail region are due to the absence of the tail section (vertical and horizontal stabilizers) in the modeling. The wing structure was also not included in the modeling. Although the tail section does contribute significantly in the pattern structure toward the back region, it was neglected here to simplify the problem since it was also a region of secondary importance for pattern coverage during landing. It is quite evident from the patterns included in Figs. 5 and 6 that the vertically-polarized aperture again provides a much wider coverage in the forward direction and it almost meets the pattern coverage requirements of 20° above the nose and 30° below the nose set forth as tentative requirements for the primary sector elevation plane coverage for the MLS. The coverage for the vertically-polarized aperture is somewhat marginal below the nose but by considering other locations above the cockpit area it may lead to better coverage below the nose.

With the availability and verification of the analytical modeling, full scale model patterns for a Boeing 737 and 747 below the nose and above the cockpit were computed. The frequency of

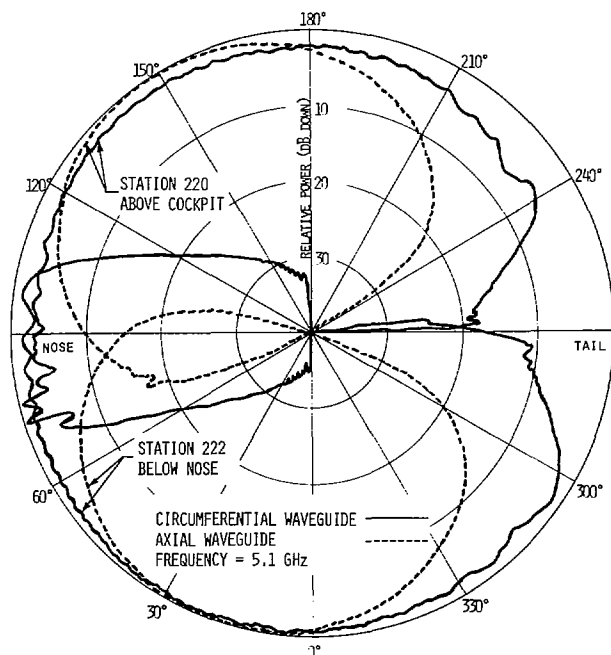


Fig. 7. Computed elevation plane amplitude patterns of circumferential and axial slot at locations 1 and 2 of Fig. 1(a) of full scale Boeing 737.

operation was 5.1 GHz, which is within the band under consideration for the MLS. The patterns below and above the nose for the Boeing 737 are shown in Fig. 7 and for the Boeing 747 in Fig. 8. These two locations were considered because they showed the most promise for forward coverage. It is quite evident that the patterns on the full scale Boeing 737 are quite similar to those of the scaled-models except for some difference in ripple structure and coverage in blind regions. These are mainly attributed to the difference in the electrical dimensions of the structure. The overall characteristics of the pattern however are mainly controlled by the shape of the structure to

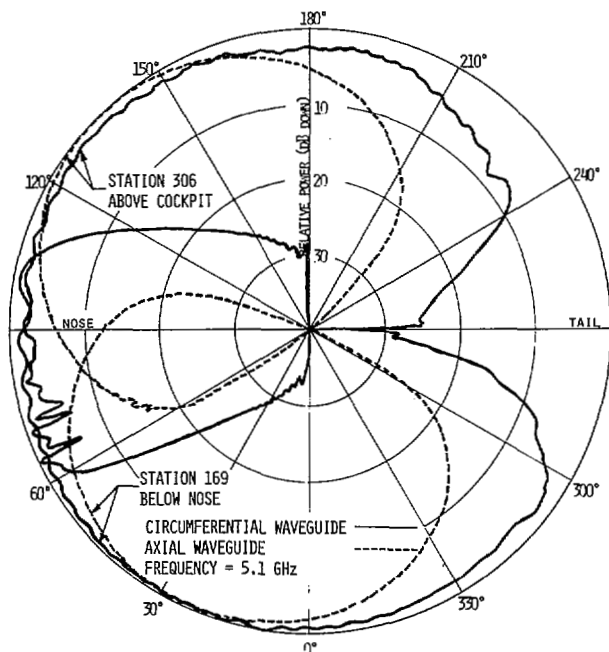


Fig. 8. Computed elevation plane amplitude patterns of circumferential and axial slot at station 169 below nose and station 306 above cockpit of full scale Boeing 747.

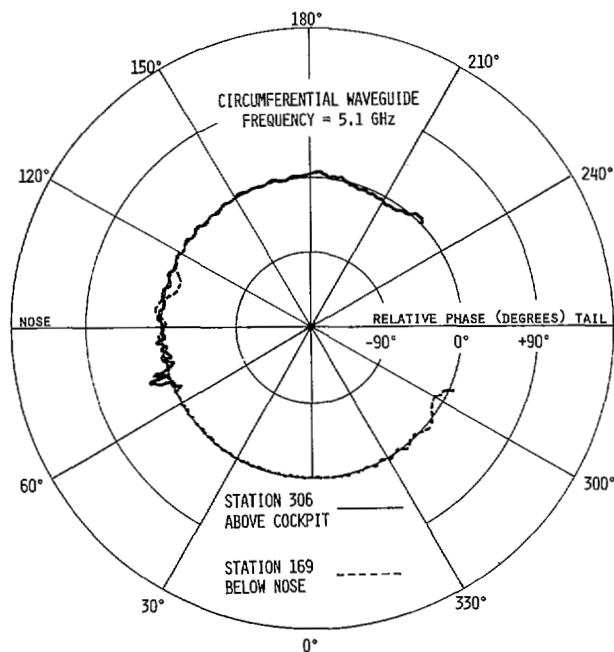


Fig. 10. Computed elevation plane relative phase patterns of circumferential slot at station 169 below nose and station 306 above cockpit of full scale Boeing 747.

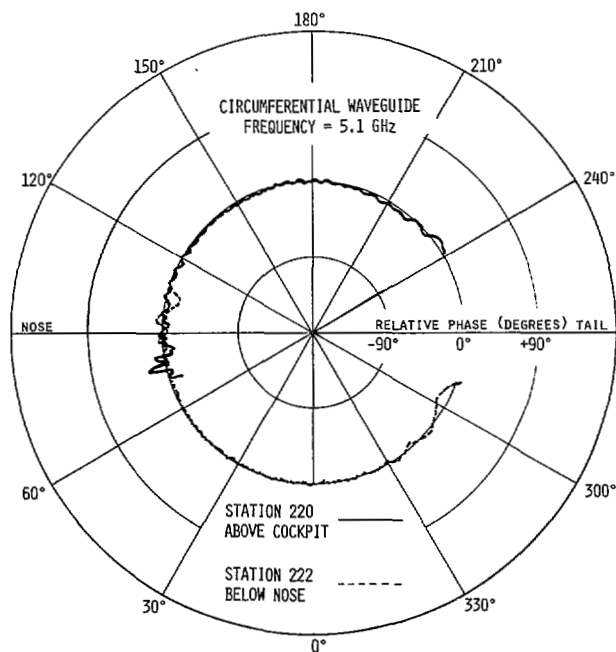


Fig. 9. Computed elevation plane relative phase patterns of circumferential slot at locations 1 and 2 of Fig. 1(a) of full scale Boeing 737.

which the antenna is mounted. The same would be true for the Boeing 747 and any other aircraft. It is observed that each of the two antenna positions on the Boeing 747 provide a much wider coverage in the forward (nose) sector than the corresponding positions for the Boeing 737, and the vertically-polarized aperture almost meets the MLS primary sector elevation plane coverage requirements of 20° above and 30° below the nose. The wider coverage provided by the Boeing 747 is attributed to the steeper slope (compared to the Boeing 737) of the aircraft frame profile at the corresponding antenna positions [10].

Another parameter of interest in the development of the MLS is the phase distortion or phase center movement because of

pattern distortion. Shown in Fig. 9 are the phase patterns of the vertically-polarized apertures positioned below the nose and above the cockpit on the full scale Boeing 737 operating at 5.1 GHz. These two positions and polarization were selected because they exhibited the most encouraging coverage for the MLS. The plotted phase patterns represent the difference in pattern when the antenna is mounted on an infinite ground plane and when it is mounted on the actual aircraft. Again the phase distortion in the forward (nose) direction is small and most of the distortion is directed toward regions of secondary importance. The phase patterns for the vertically-polarized aperture below the nose and above the cockpit for the Boeing 747 are shown in Fig. 10.

IV. CONCLUSIONS

Analytical radiation models of antennas mounted at different locations along an aircraft structure have been developed and compared with scaled-model measurements. This agreement is very good considering the complexity of the problem.

Radiation pattern coverage in the elevation plane for vertically- and horizontally-polarized antennas on scaled and full scale models has been examined with MLS requirements as criteria. It is quite evident that, for flush mounted antennas, elevation plane vertically-polarized antennas located above the cockpit and below the nose of the aircraft exhibit the most promising characteristics in the elevation plane. From the viewpoint of landing it would seem that antennas mounted above the cockpit would be more attractive because of the absence of any structural features within the airframe during landing, such as landing gear, which would distort the pattern.

V. ACKNOWLEDGMENT

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Cross Polarization in Cassegrainian and Front-Fed Paraboloidal Antennas

MEHMET SAFAK AND PAUL P. DELOGNE

Abstract—This paper presents the formulas relating feed polarization to aperture plane polarization, shows that the cross polarization in reflector antennas is closely related to the feed design, gives the ideal feed polarizations yielding zero aperture cross-polarized fields, and also puts an end to the controversy recently arisen about the relative susceptibility of Cassegrainian and front-fed antennas to depolarization effects.

INTRODUCTION AND DEFINITIONS

Frequency reuse by means of cross-polarized channels is intended for future satellite communication systems. One of the most important problems with respect to this application is the design of reflector antennas with a very high polarization purity, at least over the main lobe.

First of all, the polarization purity of an antenna must be clearly defined, even for a single polarization antenna. For doing this, it is necessary to define in each radial direction a nominal polarization. Considering the spherical coordinate system, the nominal polarization in a given direction (θ, ϕ) is defined by a unit complex vector in the base subtended by the unit vectors $(\bar{u}_\theta, \bar{u}_\phi)$, say

$$\bar{e}_n(\theta, \phi) = e_{n\theta}\bar{u}_\theta + e_{n\phi}\bar{u}_\phi \quad (1)$$

where $e_{n\theta}$ and $e_{n\phi}$ are given functions of (θ, ϕ) , with

$$|\bar{e}_n|^2 = |e_{n\theta}|^2 + |e_{n\phi}|^2 = 1. \quad (2)$$

Of course, it is immediately possible to define in the base $(\bar{u}_\theta, \bar{u}_\phi)$, a unit vector

$$\bar{e}_\perp(\theta, \phi) = e_{\perp\theta}\bar{u}_\theta + e_{\perp\phi}\bar{u}_\phi \quad (3)$$

which is orthogonal to $\bar{e}_n$ for all radial directions, i.e.,

$$\bar{e}_\perp \cdot \bar{e}_n^* = 0. \quad (4)$$

More specifically, defining a complex angle ψ by

$$\tan \psi = \frac{e_{n\phi}}{e_{n\theta}} \quad (5)$$

and writing

$$\bar{e}_n = e_{n\theta}(\bar{u}_\theta + \tan \psi \bar{u}_\phi) \quad (6)$$

one has, apart from an arbitrary phase,

$$\bar{e}_\perp = e_{n\theta} \tan \psi (\bar{u}_\theta - \cot^* \psi \bar{u}_\phi). \quad (7)$$

Now, let the fields radiated by the antenna be

$$\begin{aligned} \bar{E}(R\bar{u}_R) &= \frac{e^{-jkR}}{R} \bar{F}(\theta, \phi) \\ &= \frac{e^{-jkR}}{R} (F_n \bar{e}_n + F_\perp \bar{e}_\perp) \end{aligned} \quad (8)$$

the cross polarization of the antenna is defined as

$$C(\theta, \phi) = -20 \log \left| \frac{F_\perp}{F_n} \right|. \quad (9)$$

This definition also applies to double polarization antennas; the two nominal polarizations should then obviously be taken orthogonal, i.e., $\bar{e}_n$ and $\bar{e}_\perp$.

CHOICE OF THE NOMINAL POLARIZATION

In a specific application to a very directive antenna, the choice of the nominal polarization depends, among other factors, on the main type of antenna misalignment or on the variation of arrival angle that can occur. Right- or left-handed circular polarization in all directions is of course ideal with respect to roll (rotation about broadside), pitch or yaw misalignment, but can be difficult to achieve. If roll is not important, linear polarization in broadside direction is useful, but what should be the nominal polarization in other directions?

Considering the z axis as broadside and, for that particular direction a nominal polarization parallel to the x axis, the following definitions have been proposed by Ludwig [1].

1) A nominal polarization identical to that of a Hertz-dipole parallel to the x axis, that is,

$$\bar{e}_n = \frac{\cos \theta \cos \phi \bar{u}_\theta - \sin \phi \bar{u}_\phi}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}}. \quad (10)$$

In this case,

$$\bar{e}_\perp = \frac{\sin \phi \bar{u}_\theta + \cos \theta \cos \phi \bar{u}_\phi}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}}. \quad (11)$$

2) A nominal polarization identical to that of a Huygens source with electric field parallel to the x axis, that is,

$$\bar{e}_n = \cos \phi \bar{u}_\theta - \sin \phi \bar{u}_\phi. \quad (12)$$

In this case,

$$\bar{e}_\perp = \sin \phi \bar{u}_\theta + \cos \phi \bar{u}_\phi. \quad (13)$$

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M. Safak was with the Telecommunications and Microwaves Laboratory, Louvain University, Louvain-la-Neuve, Belgium. He is now with the Electrical Engineering Department, Technological University, Eindhoven, The Netherlands.

P. P. Delogne is with the Telecommunications and Microwaves Laboratory, Louvain University, Louvain-la-Neuve, Belgium.